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EFFECT OF SURFACE ROUGHNESS ON KELVIN-HELMHOLTZ INSTABILITY IN PRESENCE OF MAGNETIC FIELD

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ABSTRACT

We study the effect of surface roughness on Kelvin-Helmholtz instability (KHI) in a fluid layer above by a porous layer and below by a rigid surface in presence of transverse magnetic field. A simple theory based on fully developed flow approximations is used to derive the dispersion relation with surface roughness for the growth rate of KHI. We replace the effect of boundary layer with Beavers and Joseph slip condition as well as roughness condition at the rigid surface. The dispersion relation is derived using suitable boundary and surface conditions and results are discussed graphically. The magnetic field is found to be stabilizing and the influence of the various parameters involved in the problem on the interface stability is thoroughly analyzed.

KEYWORDS: KHI, magnetic field, BJ-slip condition, porous layer, dispersion relation, surface roughness.

INTRODUCTION

Kelvin-Helmholtz instability is one of the basic instabilities of two-fluid systems, which affects an interface. In Engineering, Kelvin -Helmholtz instability plays an essential role in transition from stratified to slug flow in horizontal pipes explored by Simmons [2]. Lord Kelvin first examined Kelvin-Helmholtz instability in 1910. An inviscid linear analysis of the phenomenon, which is applicable in case of two liquids with similar densities, can be found in various textbooks, for example in Chandrasekhar [3], and Drazin & Reid [4]. The problem becomes much more complicated for large density differences, which appears in case of liquid and gas. For example the instability of sea surface appears at wind speeds significantly lower than the critical wind speed given by linear inviscid analysis Gondret & Rabaud [5]. This phenomenon called "subcritical" Kelvin-Helmholtz instability (with high density difference) was analyzed by Meignin [6] and was found to be result of nonlinear analysis.

Kelvin-Helmholtz instability appears in stratified two-fluid flows, in the presence of a small disturbance and relative velocity that is larger than critical. The disturbance causes change of the velocity field. Because of the continuity equation, the velocity of one fluid increases and of the other one decreases. The change in velocity field changes

pressure (Bernoulli's equation). Pressure force is increasing the disturbance; surface tension force and gravity force are decreasing the disturbance. If the pressure force is larger than the sum of surface tension and gravity forces, the Kelvin-Helmholtz instability occurs. A linear theory of the KHI for parallel flow in porous media was introduced by Bau[7] for the Darcian and non-Darcian flows. In both cases, Bau found that the velocities should exceed some critical value for the instability to manifest itself. The instability of plane interface between two uniform superposed fluids through a porous medium was investigated by Kumar [8]. They used linear stability analysis to obtain a characteristic equation for the growth of the disturbance.

The nonlinear Kelvin -Helmholtz instability of a horizontal interface between a magnetic inviscid incompressible liquid and an inviscid laminar subsonic magnetic gas is investigated in the presence of a normal magnetic field by Zakaria[9]. El-Sayed [10] investigated the RTI problem of rotating stratified conducting fluid layer through porous medium in the presence of an inhomogenous magnetic field. This problem corresponds physically (in astrophysics) to the RTI of an equatorial section of a planetary magnetosphere or of stellar atmosphere when rotation and magnetic field are perpendicular to

gravity. The KHI of two superposed viscous fluids in a uniform vertical magnetic field is discussed in the presence of effects of surface tension and permeability of porous medium by Bhatia and Sharma [11]. Following Babchin et al., [12] and Rudraiah et al., [13], a simple theory based on Stokes and lubrication approximations is used in this study by replacing the effect of the boundary layer with a Beavers and Joseph [14] slip condition, with the primary objective of using porous layer to suppress the growth rate of KHI. In the above studies the fluid has been considered to be Newtonian. El-Dib and Matoog [15] have studied the Electro-rheological Kelvin-Helmholtz instability of a fluid sheet. This work deals with the gravitational stability of an electrified Maxwellian fluid sheet shearing under the influence of a vertical periodic electric field. The field produces surface charges on the interfaces of the fluid sheet. Due to the rather complicated nature of the problem a mathematical simplification is considered where the weak effects of viscoelastic fluids are taken into account. Asthana and Agrawal [16] have applied the viscous potential theory to analyze Kelvin-Helmholtz instability with heat and mass transfer and observed that heat and mass transfer has destabilizing effect on relative velocity when lower fluid viscosity is low while it has stabilizing effect when lower fluid viscosity is high. Khalil Elcoot [17] has studied the new analytical approximation forms for non-linear instability of electric porous media. In this work, we have examined the effects of stability of the normal electric field on the porous media, in view of the non-linear theory. The main purpose is to discuss modulation instability of a finite wavetrain solution by using the method of multiple scales perturbation, and comparing the results with the linear instability theory. Chavaraddi et al., [18] have studied the electrohydrodynamic Kelvin-Helmholtz instability in a fluid layer bounded above by a porous layer and below by a rigid surface. Recently, Chavaraddi et al [19] have studied the Kelvin-Helmholtz discontinuity between two viscous conducting fluids in a transverse magnetic field through a porous medium in the presence of the effects of surface tension using B-J condition [14] at the interface. The objective of this paper is to predict the effect of surface roughness at the boundary formulated by Miksis and Devis [20] on Kelvin-Helmholtz instability.

The paper is organized as follows. The basic equations are established in section 2 together with Maxwell's equations. The basic equations are simplified and non-dimensionalized using the following Stokes and lubrication approximations in this section. The resulting dispersion relation is derived using suitable boundary and surface conditions in section 3. The cutoff and maximum wave numbers and the corresponding maximum growth rate are also obtained in section 3. The results are discussed and some important conclusions are drawn in final section of this paper.

MATHEMATICAL FORMULATION

The physical configuration is shown in Figure 1. We consider a thin target shell in the form of a thin film of unperturbed thickness h (Region 1) filled with an incompressible, viscous, poorly electrically

conducting light fluid of density ρ_f bounded below

by a rigid surface at $y=0$ and above by an incompressible, viscous poorly conducting heavy fluid of density ρ_p saturating a dense porous layer

of large extent compared to the shell thickness h . The co-ordinates x and y spans the horizontal and vertical directions. The interfacial $y=h$ is denoted by $\eta(x,t)$.

When the interface is flat then $\eta = 0$ when $y=h$. The

fluid velocity vector $\vec{q} = (u, v)$ and the fluid is assumed to be Newtonian, viscous electrically conducting and incompressible. The viscosity of fluid (porous medium) is given by $\mu_f(\mu_p)$, ε the porous parameter, κ the permeability of the porous medium and α is the slip parameter at the interface.

The stress gradient δ is related to the gravitational acceleration through the relation $\delta = g(\rho_p - \rho_f)$.

The perturbed interface $\eta(x,t)$ is along the y direction.

The basic equations for clear fluid layer (region 1) and those for porous layer (region 2) are as given below:

Region-1:

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

$$\rho_f \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = -\nabla p + \mu_f \nabla^2 \vec{q} + \mu_0 (\vec{J} \times \vec{H}) \tag{2.2}$$

Maxwell's Equations:

$$\nabla \cdot \vec{E} = 0, \nabla \cdot \vec{H} = 0, \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{2.3}$$

and the auxiliary equations

$$\vec{D} = \epsilon_0 \vec{E}, \vec{B} = \mu_0 \vec{H}, \vec{J} \times \vec{B} = \sigma [\vec{E} + \vec{q} \times \vec{B}] \times \vec{B} \tag{2.4}$$

Region-2:

$$Q = -\frac{k}{\mu} \frac{\partial p}{\partial x} \tag{2.5}$$

where $\vec{q} = (u, v)$ the fluid velocity, \vec{E} the electric field, \vec{H} the magnetic field, \vec{J} the current density, \vec{D} the dielectric field, \vec{B} the magnetic induction, σ the electrical conductivity, k the permeability of the porous medium, p the pressure, μ_0 magnetic permeability, $\vec{Q} = (Q, 0, 0)$ the uniform Darcy velocity, μ the fluid viscosity and ρ the fluid density.

The basic equations are simplified using the following Stokes and lubrication and electrohydrodynamic approximations (See Rudraiah et al[13]):

- (i) The electrical conductivity of the liquid, σ , is negligibly small, i.e., $\sigma \ll 1$.
- (ii) The film thickness h is much smaller than the thickness H of the dense fluid above the film. That is $h \ll H$
- (iii) The surface elevation η is assumed to be small compared to film thickness h . That is $\eta \ll h$
- (iv) The Strouhal number S , a measure of the local acceleration to inertial acceleration in Eq. (2.2), is negligibly small.
That is

$$S = \frac{L}{TU} \ll 1$$

where $U = v/L$ is the characteristic velocity, ν the kinematic viscosity, $L = \sqrt{\gamma/\delta}$ the characteristic length and $T = \mu\gamma/h^3\delta^2$ the characteristic time.

Under these approximations Eqs.(2.1) and (2.2) for fluid in the film, after making dimensionless using

$$u^* = \frac{u}{\delta h^2 / \mu_f}, v^* = \frac{v}{\delta h^2 / \mu_f}, p^* = \frac{p}{\delta h}, Q^* = \frac{Q}{\delta h^2 / \mu_f}, t^* = \frac{t}{\delta h / \mu_f}, x^* = \frac{x}{h}, y^* = \frac{y}{h} \tag{2.6}$$

become (after neglecting the asterisks for simplicity)

Region 1:

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{2.7}$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{2.8}$$

$$0 = -\frac{\partial p}{\partial y} \tag{2.9}$$

where $M = \mu_0 H_0 h \sqrt{\sigma_f / \mu_f}$ is the Hartmann number which is the ratio of Darcy resistance to the viscous force.

Region 2:

$$Q = -\frac{1}{\sigma_p^2} \frac{\partial p}{\partial x} \tag{2.10}$$

where

$$\sigma_p = h / \sqrt{k} \text{ is the porous parameter.}$$

DISPERSION RELATION

To find the dispersion relation, first we have to find the velocity distribution from Eq. (2.8) using the following boundary and surface conditions:

$$-\beta \frac{\partial u}{\partial y} = u \quad \text{at } y = 0 \tag{3.1}$$

$$\frac{\partial u}{\partial y} = -\alpha_p \sigma_p (u_B - Q) \quad \text{at } y = l \tag{3.2}$$

where

$$u = u_B \quad \text{at } y = l$$

$$v = \frac{\partial \eta}{\partial t} \quad \text{at } y = l \tag{3.3}$$

$$p = -\eta - \frac{1}{B} \frac{\partial^2 \eta}{\partial x^2} \quad \text{at } y = l. \tag{3.4}$$

Here $B = \delta h^2 / \gamma$ is the Bond number, β is the roughness parameter and $\eta = \eta(x, y, t)$ is the elevation of the interface.

The solution of (2.8) subject to the above conditions is

$$u = P \left[a_1 \text{Cosh}My + a_2 \text{Sinh}My - \frac{1}{M^2} \right] \tag{3.5}$$

where

$$a_1 = \frac{1 - a_2 \beta M^2 \text{Cosh}M}{M^2 \text{Cosh}M}$$

$$a_2 = - \frac{\left[\frac{\tanh M}{M} + \frac{\alpha_p}{\sigma_p} \right]}{M[\text{Cosh}M - \beta \text{Sinh}M] + \alpha_p \sigma_p [\text{Sinh}M - \beta \text{Cosh}M]}$$

$$p = \frac{\partial p}{\partial x}.$$

After integrating Eq.(2.7) with respect to y between $y = 0$ and 1 and using Eq.(3.5), we get

$$v(1) = \left[\frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B} \frac{\partial^4 \eta}{\partial x^4} \right] \Delta_2 \tag{3.6}$$

where

$$\Delta_2 = \frac{a_1 M \text{Sinh}M + a_2 M (\text{Cosh}M - 1) - 1}{M^2}.$$

Then Eq.(3.3), using Eqs.(3.6) and (3.4), becomes

$$\frac{\partial \eta}{\partial t} = \left[\frac{\partial^2 \eta}{\partial x^2} + \frac{1}{B} \frac{\partial^4 \eta}{\partial x^4} \right] \Delta_2. \tag{3.7}$$

To investigate the growth rate, n , of the periodic perturbation of the interface, we look for the solution of Eq.(3.7) in the form

$$\eta = \eta(y) \exp\{i\ell x + nt\} \tag{3.8}$$

where ℓ is the wave number and $\eta(y)$ is the amplitude of perturbation of the interface.

Substituting Eq.(3.8) into (3.7), we obtain the dispersion relation in the form

$$n = \ell^2 \left(1 - \frac{\ell^2}{B} \right) \Delta_2. \tag{3.9}$$

Also, Eq. (3.9) can be expressed as

$$n = n_b - \ell \beta v_a \tag{3.10}$$

where

$$n_b = \frac{\ell^2}{3} \left[1 - \frac{\ell^2}{B} \right], \quad \beta = \Delta_2 \ell \left[1 - \frac{\ell^2}{B} \right], \quad v_a = \left(\frac{1 - 3\Delta_2}{3\Delta_2} \right) \left(1 - \frac{\ell^2}{B} \right).$$

Setting $n = 0$ in Eq.(3.9), we obtain the cut-off wavenumber, ℓ_{ct} in the form

$$\ell_{ct} = \sqrt{B} \tag{3.11}$$

because ℓ and Δ_2 are non-zero.

The maximum wavenumber, ℓ_m obtained from Eq.(3.9) by setting $\frac{\partial n}{\partial \ell} = 0$ is

$$\ell_m = \sqrt{\frac{B}{2}} = \frac{\ell_{ct}}{\sqrt{2}} \tag{3.12}$$

because ℓ and Δ are different from zero.

The corresponding maximum growth rate, n_m , is

$$n_m = \frac{B}{4} \Delta_2 \tag{3.13}$$

Similarly, using $\ell_m = \sqrt{B/2}$, we obtain

$$n_{bm} = \frac{B}{12} \quad (3.14)$$

and hence

$$G_m = \frac{n_m}{n_{bm}} = 3\Delta_2. \quad (3.15)$$

The growth rate given by Eq.(3.9) is computed numerically for different values of physical parameters which involved in the problem and the results are presented graphically in Figures 2-5.

RESULTS AND DISCUSSION

In this study we have shown the surface instability of KH type in a fluid layer bounded above by a porous layer and below by a rigid surface is affected by the effect of magnetic field and surface roughness. Numerical calculations were performed to determine the growth rate at different wavenumbers for various fluid properties like Hartmann number M , Bond number B , porous parameter σ_p and roughness parameter β . We have plotted the dimensionless growth rate of the perturbation against the dimensionless wavenumber for some of the cases only.

When we fix all the input parameters except the ratio of the Hartmann number M , we find that the higher the Hartmann ratio the more stable the interface is. In Figure 2, we have plotted the the growth rate against the wavenumber in the case where $\alpha_p = 0.1$, $B = 0.02$, $\sigma_p = 4$ and $\beta = 3.3 \times 10^{-3}$ for different values of the Hartmann number M . Increasing the Hartmann ratio results is slightly increasing the critical wavenumber and decreasing the maximum growth rate. Thus it has a stabilizing effect for the selected values of input parameters due to the increased in Hartmann ratio (Lorentz force to viscous force).

In addition, we have investigated the effect of the surface tension of the fluid on the instability of the interface. In our sample calculations, we have taken $\alpha_p = 0.1$, $M = 2$, $\sigma_p = 4$ and $\beta = 3.3 \times 10^{-3}$ with the variation of Bond number B . For this input parameters, the critical wavenumber and maximum growth rate decreased as the ratio of the Bond number B decreased from 0.4 to 0.1 as observed in Figure 3. This is because the Bond number is reciprocal of surface tension and thus showing that an increase in surface tension decreases the growth rate and hence make the interface more stable.

However, in order to understand the effect of the porous properties on the instability, we now fix values of other parameters $\alpha_p = 0.1$, $B = 0.02$, $M = 2$ and $\beta = 3.3 \times 10^{-3}$ and vary the ratios of the porous parameters. Figure 4 displays the results of our calculations, showing that increasing the ratio of porous parameters σ_p from 4 to 100 (and thus increasing the Darcy resistance compared to the viscous force) increases the critical wavelength and increases the maximum growth rate, thus having a destabilizing effect by this parameter. We conclude that an increase in σ_p also destabilize the KHI.

Finally, we have fixed the values of the other parameters $\alpha_p = 0.1$, $B = 0.02$, $M = 2$ and $\sigma_p = 4$ with variation of the roughness parameter β as shown in Fig.5. It is clear that an increase in surface roughness parameter is to decrease in the growth rate of the interface; this is because the resistance offered by the surface roughness should be overcome, in that process a part of kinetic energy is converted into potential energy. Hence the effect of surface roughness is to reduce the growth rate of the interface and thus to make the system stable.

CONCLUSION

We have studied the linear stability of a two-fluid flow in a channel where the fluids are assumed to be Newtonian with different fluid properties (Hartmann number, Surface tension, porous parameter and surface roughness) and subjected to magnetic field normal to their interface with surface roughness. For this purpose, we have derived and then linearized the equations of motion where the interaction between the hydrodynamic and electric problems occurs through the stress balance at the fluid interface. The growth rate of the perturbation was then computed by using the normal mode method and its variation studied as a function of the dimensionless parameter

Hartmann M, Bond number B and porous parameter σ_p in addition to roughness parameter β . While two layer flows in channels of small dimensions are rather stable, the instability of the fluid-porous interface is highly desirable in certain cases, particularly for chemical industry, in petroleum production engineering applications where the mixing of reagents are crucial steps in the process. However, in systems of larger scale, the instability of the fluid-porous interface in a channel is often an undesired physical phenomenon. In such situations, controlling the flow requires the stabilization of the interface. In searching for a method capable of either stabilizing a potentially unstable interface or destabilizing a potentially stable one, we have investigated the role of the magnetic field on the two-layer channel flow problem in presence of surface roughness; demonstrated that either destabilization or stabilization can be obtained and presented growth rates in situations where the magnetic field is stabilizing or destabilizing over a broad range of wavenumbers for increasing in Hartmann number M. But in the case of variation in Bond number is to increase in surface tension decreases the growth rate and hence make the interface more stable. Also we conclude that the increase in the porous parameter is to increase the growth rate showing thereby the destabilizing effect on the interface.

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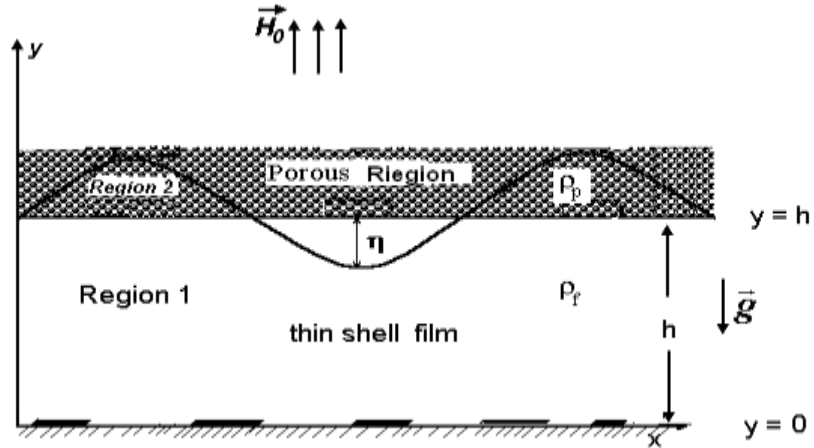


Figure 1: Physical Configuration

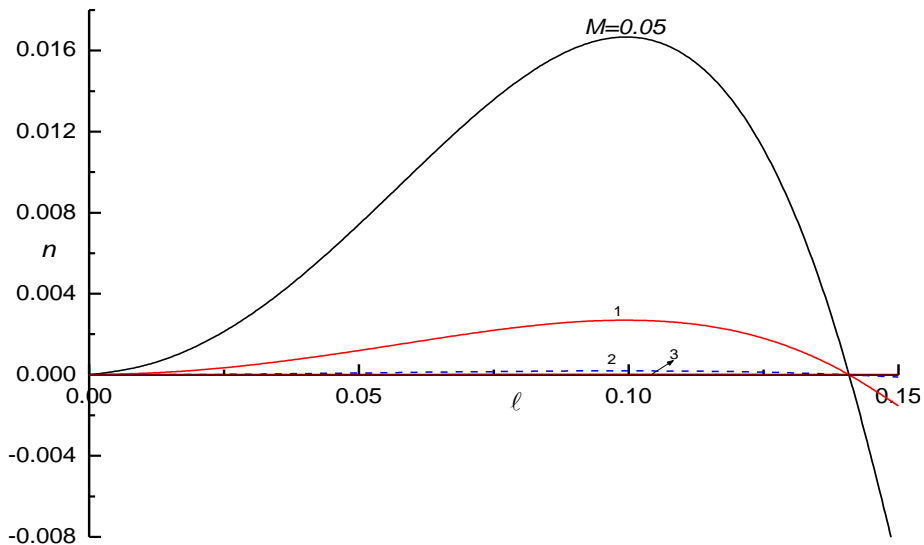


Figure 2: Growth rate, n versus the wavenumber, ℓ for different values of Hartmann number, M when $\alpha_p = 0.1$, $B = 0.02$, $\beta = 3.3 \times 10^{-3}$ and $\sigma_p = 4$.

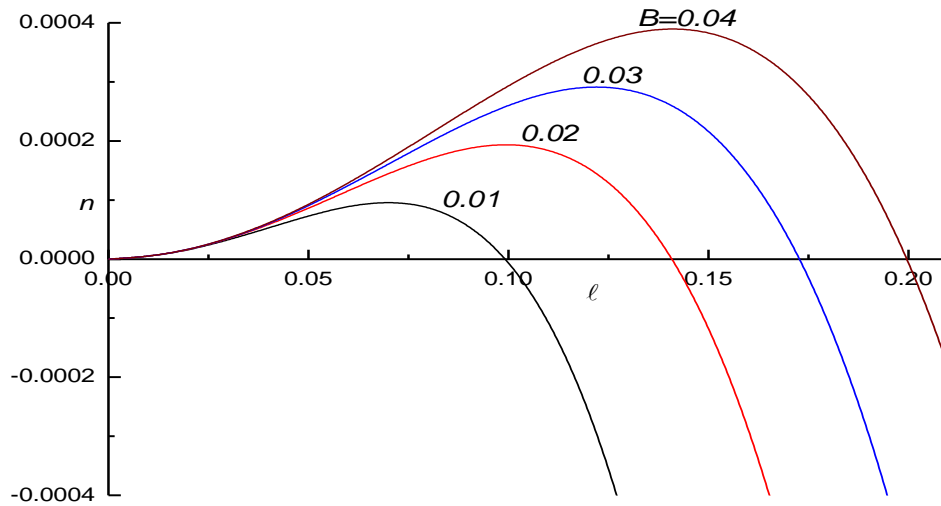


Figure 3: Growth rate, n versus the wavenumber, ℓ for different values of Bond number, B when $\alpha_p = 0.1, M = 2, \sigma_p = 4$ and $\beta = 3.3 \times 10^{-3}$.

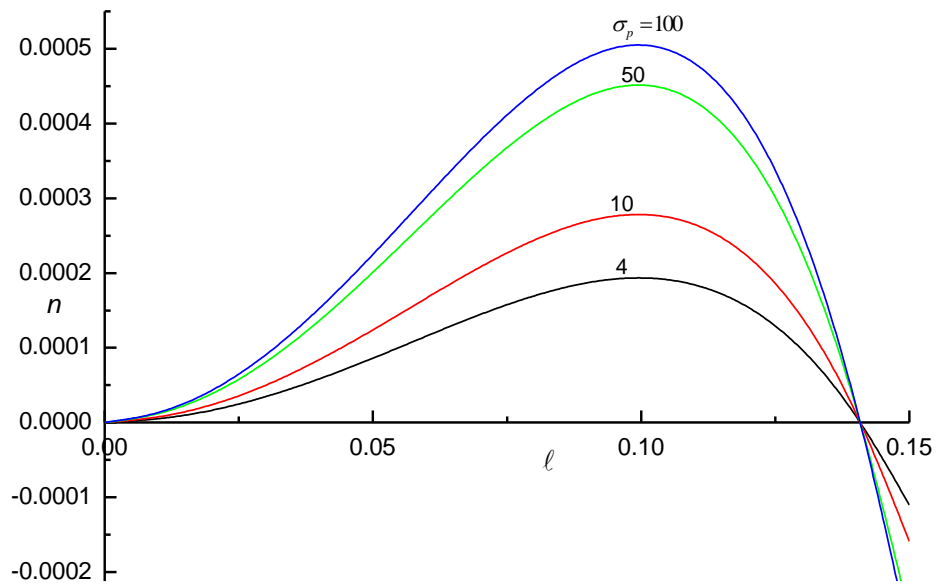


Figure 4: Growth rate, n versus the wavenumber, ℓ for different values of Porous parameter, σ_p when $\alpha_p = 0.1, B = 0.02, \beta = 3.3 \times 10^{-3}$ and $M = 2$.

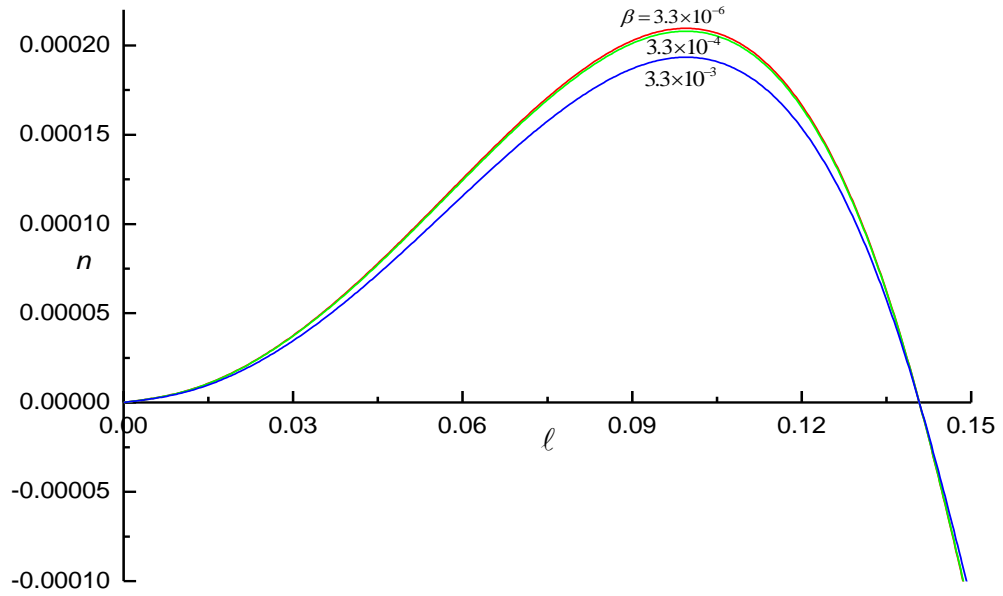


Figure 5: Growth rate, n versus the wavenumber, ℓ for different values of Roughness parameter, when $\alpha_p = 0.1$, $B = 0.02$, $\sigma_p = 4$. $\beta = 3.3 \times 10^{-3}$ and $M = 2$.